دانشگاه صنعتی سهند، تبریز، ایران
دوازدهمین سمینار بین المللی
جبر خطی
و کاربردهای آن
۲۷ الی ۲۸ تیرماه ۱۴۰۲
Sahand University of Technology, Tabriz, Iran
The 12th International Seminar on
Linear Algebra
and its Applications
July 18 - 19, 2023

Fourier Like Systems, Frame of Translates and their Oblique Duals on LCA-groups

R. A. Kamyabi Gol*

Department of Mathematics, Ferdowsi University of Mashhad , Mashhad, Iran

Abstract

The theory of frames of translates has an essential role in many areas of mathematics and its applications such as wavelet theory and reconstruction of signals from sample values $[1-4, 6, 11, 12, 13]$. A lattice system of translates is a sequence in $L^2(\mathbb{R})$ that has the form $\mathcal{T}(g) = \{g(. - ak)\}_{k\in\mathbb{Z}}$ where $g \in L^2(\mathbb{R})$ and $a > 0$ are fixed. In the setting of $L^2(\mathbb{R})$, it is known that frames of translates can be characterized in terms of a 1-periodic function ([3, 6]). More precisely, for $g \in L^2(\mathbb{R})$, if we define $\Phi_g(\omega) = \sum_{k\in\mathbb{Z}} |\widehat{\phi}(\omega+k)|^2$, then Φ_g is a 1-periodic function which characterizes frames of translates as follows.

(a) $\mathcal{T}(g)$ is a frame sequence if and only if there exist $0 < A \leq B < \infty$ such that $A \leq \Phi_g \leq B$, a.e. on the zero set of Φ_g.

(b) $\mathcal{T}(g)$ is a Riesz basis for the closure span of $\mathcal{T}(g)$ if and only if there exist $0 < A \leq B < \infty$ such that $A \leq \Phi_g \leq B$, a.e.

(c) $\mathcal{T}(g)$ is an orthonormal basis for the closure span of $\mathcal{T}(g)$ if and only if $\Phi_g = 1$ a.e.

Our goal in this presentation is a generalization of frames of translates in the setting of locally compact abelian groups. Let G be a locally compact abelian (LCA) group and Γ be a uniform lattice in G (i.e. a discrete subgroup of G which is co-compact), with the annihilator Γ^* in $\widehat{G}$ (the dual group of G)$[5, 7, 8, 10, 14-16]$. For $g \in L^2(G)$, a system of translates generated by g via Γ, is defined as

$$\mathcal{T}(g) = \{g(. + \gamma)\}_{\gamma\in\Gamma}$$

We define a Γ^*-periodic function Φ_g on $\widehat{\Gamma}$ and investigate a characterization of translates of $g \in L^2(G)$ to have some properties. We achieve our goal by using an isometry from $L^2(G)$ into $L^2(\widehat{\Gamma})$, in such a way that the system of translates in $L^2(G)$ is transferred to a nice Fourier-like system in $L^2(\widehat{\Gamma})$. To do so, we consider a fix $\varphi \in L^2(\widehat{\Gamma})$ and define the Fourier-like system generated by φ as $\mathcal{E}(\varphi) = \{X_\gamma\varphi\}_{\gamma\in\Gamma}$, where X_γ is the corresponding character γ on $\widehat{\Gamma}$. We deduce the structure of the canonical dual frame of a frame sequence $\mathcal{T}(g)$. Using the fact that the frame operator of a frame of translates commutes with the translation operator, it is shown that the canonical dual frame of $\mathcal{T}(g)$ has the same form $\mathcal{T}(h)$ for some $h \in \overline{\text{span}}(\mathcal{T}(g))$. Some properties of Φ_g which are useful in the study of the translates sequence generated by g are

*Speaker. Email address: kamyabium.ac.ir

investigated. In particular, it is shown that if Φ_g is continuous, then $\mathcal{T}(g)$ can not be a redundant frame.

Keywords: locally compact abelian group, Fourier-like system, Fourier-like frame, frame of translates, oblique dual.

Mathematics Subject Classification [2010]: Primary 47A15 ; Secondary 42B99, 22B99.

References

[1] C. Cabrelli, C. Mosquera and V. Patemostro, Linear combinations of frame generators in systems of translates, *J. Math. Anal. Appl.*, 413 (2014), 776–788.

[2] P. Casazza, O. Christensen and N.J. Kalton, Frames of translates, *Collect. Math.*, (2001), 35–54.

[3] O. Christensen, *An Introduction to Frames and Riesz Bases*, Birkhauser, 2015.

[4] C. Demeter, Linear independence of time frequency translates for special configurations, *Math. Res. Lett.*, 17 (2010), 761–779.

[5] G.B. Folland, *A Course in Abstract Harmonic Analysis*, CRS Press, 1995.

[6] C. Heil, *A Basis Theory Primer*, Birkhauser, 2011.

[7] E. Hewitt and K.A. Ross, *Abstract Harmonic Analysis*, vol. 1, Springer-Verlag, 1963.

[8] R.A. Kamyabi Gol and R. Raisi Tousi, Bracket products on locally compact abelian groups, *J. Sci. Islam. Repub. Iran*, 19 (2008), No. 2, 153–157.

[9] H.O. Kim and J.K. Lim, New characterizations of Riesz bases, *Appl. Comput. Harmon. Anal.*, 4 (1997), 222–229.

[10] G. Kutyniok, Time frequency analysis on locally compact groups, Ph.D thesis, Padeborn University, 2000.

[11] M. Nielsen and H. Sikie, Schauder bases of integer translates, *Appl. Comput. Harmon. Anal.*, 23 (2007), 259–262.

[12] A. Olevskii and A. Ulanavskii, Almost linear translates. Do nice generators exist?, *J. Foutier Anal. Appl.*, 10 (2004), 93–104.

[13] T.E. Olson and R.A. Zalik, *Nonexistence of a Riesz basis of translates, in: Approximation Theory*, Lecture Notes in Pure and Applied Math. Vol. 138, Dekker, New York, 1992, 401–408.

[14] R. Raisi Tousi, Shift invariant spaces, MRA and bracket products on LCA groups, Ph.D. thesis, Ferdowsi University of Mashhad, 2008.

[15] A. Safapour and R.A. Kamyabi Gol, A necessary condition for Weil-Heisenberg frames, *Bull. Iranian Math. Soc.*, 2 (2004), 67–79.

[16] N. Seyedi and R.A. Kamyabi Gol , On the frames of translates on locally compact abelian groups, *Bull. Iranian Math. Soc.*, (2021), 1–22.