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Ballantine's type theorem for complex symplectic group[1]

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Abstract

In the late 1960 Ballantine showed that every matrix with positive determinant is a product of five positive definite matrices.

We consider the complex symplectic group $\mathrm{Sp}(2n,\mathbb{C})$:

$$\mathrm{Sp}(2n,\mathbb{C}) = \left\{A \in \mathrm{GL}(2n,\mathbb{C}) : A^\top J_n A = J_n\right\},$$

where

$$J_n = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix}.$$

The symplectic group is a classical group defined as the set of linear transformations of a $2n$-dimensional vector space over $\mathbb{C}$, which preserve the non-degenerate skew-symmetric bilinear form that is defined by J_n. We show that every symplectic matrix is a product of five positive definite symplectic matrices. We also show that five is the best in the sense that there are symplectic matrices which are not product of less.

This is a joint work with Daryl Q. Granario, De La Salle University, Philippines.

Keywords: Ballantine's theorem, Radjavi's theorem, complex symplectic group, symplectic positive definite matrices

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