دانشگاه صنعتی سهند، تبریز، ایران
دوازدهمین سمینار بین المللی
جبر خطی
و کاربردهای آن
۲۷ الی ۲۸ تیرماه ۱۴۰۲

Sahand University of Technology, Tabriz, Iran
The 12th International Seminar on
Linear Algebra
and its Applications
July 18 - 19, 2023

Positive Classes of Matrices

Kazem Ghanbari[1,2*]

[1]Department of Mathematics, Sahand University of Technology, Tabriz, Iran

[2]School of Mathematics and Statistics, Carleton University, Ottawa, Canada

Abstract

In this lecture we present different types of positivity concept in matrix analysis. Any kind of positive matrix has own typical applications. Entrywise positivity, definite positivity, complete positivity and total positivity are the main types of positivity in matrix analysis. The concept of a positive definite matrix (PD) is well-known for most people having the elementary course in linear algebra, but the other types of positivity are not quiet well-know as PD matrices, thus we present other types of positivities in matrix theory. For more complete information on PD matrices see [1]. In linear algebra any real matrix with nonnegative entries is called *Nonnegative Matrix* (NM).

A matrix which is both nonnegative and positive semi-definite is called *doubly nonnegative matrix* (DNM). The Perron–Frobenius theorem, proved by Oskar Perron (1907) and Georg Frobenius (1912), is the most important result stating that a real square matrix with positive entries has a unique largest real eigenvalue and that the corresponding eigenvector can be chosen to have strictly positive entries. This theorem has signifant applications [2, 4–6]. If a symmetric matrix A can be factorized of the form $A = BB^T$ where B is a non-negative matrix, then A is called a *Completely Matrix* (CP). Completely positive matrices have arisen in some situations in economic modelling and appear to have some applications in statistics, and they are also appear in quadratic optimisation, for more details see [3]. Any real matrix with nonnegative minors are called *Totally Non-Negative* (TN) matrix. If all minors are strictly positive then A is called *Totally Positive* (TP). This topic appears in the spectral properties of kernels of ordinary differential equations whose Green's function is totally positive (studied by M. G. Krein and some colleagues in the mid-1930s) [7–9]. In this presentation we give a detailed picture of all kinds of positivity mentioned above.

Keywords: Positivity, Total Positivity, Complete Positivity

Mathematics Subject Classification [2010]: 15A18, 15A23

References

[1] R. Bhatia, *Positive definite Matrices*, Princeton University Press 2007.

[2] Abraham Berman, Robert J. Plemmons, Nonnegative Matrices in the Mathematical Sciences, 1994, SIAM.

*Speaker. Email address: kazemghanbari@math.carleton.ca

[3] Abraham Berman, Completely Positive Matrices, World Scientific Publishing (2003)

[4] A. Graham, Nonnegative Matrices and Applicable Topics in Linear Algebra, John Wiley and Sons, New York, 1987.

[5] R. A. Horn and C.R. Johnson, Matrix Analysis, Cambridge University Press, 1990

[6] Seneta, E. Nonnegative matrices and Markov chains. 2nd rev. ed., 1981, XVI, 288 p., Softcover Springer Series in Statistics. (Originally published by Allen and Unwin Ltd., London, 1973)

[7] George M. Total Positivity, Interpolation and Approximation by Polynomials, Springer (2003)

[8] Graham Gladwell, Inverse problems in vibration, Kluwer Academic Publishers, (2004)

[9] Shaun Fallat and Charles R. Johnson, Totally Nonnegative Matrices, Princeton University Press (2011)