Sparsest Stochastic Matrices with an Eigenvalue on the boundary of the Karpelevič Region

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Abstract

A celebrated result of Karpelevič [1] describes Θ_n, the collection of all eigenvalues arising from the stochastic matrices of order n. Johnson and Paparella [2] construct, for each λ on the boundary of the Θ_n, a stochastic matrix of order n having λ as an eigenvalue.

In this talk we will first present the results from [3] where all possible stochastic realizations of an eigenvalue on the border of Θ_n are considered. After identifying all sparsest matrices with an eigenvalue on the border of the Karpelevič region, we will show when such a matrix can be written as a positive integer power of another stochastic matrix.

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References

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